Local Non-Bossiness

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What is school choice?

- **Problem:** Assign students to schools (without money).
- Centralized organization.
- Students have (ordinal) preferences over schools.
- Schools have priorities (ranking over students).
- Schools' capacities.
- Mechanism $f(Pref, Priorities, Q) \rightarrow$ Matching.

Increasing number of centralized systems

Countries with school choice

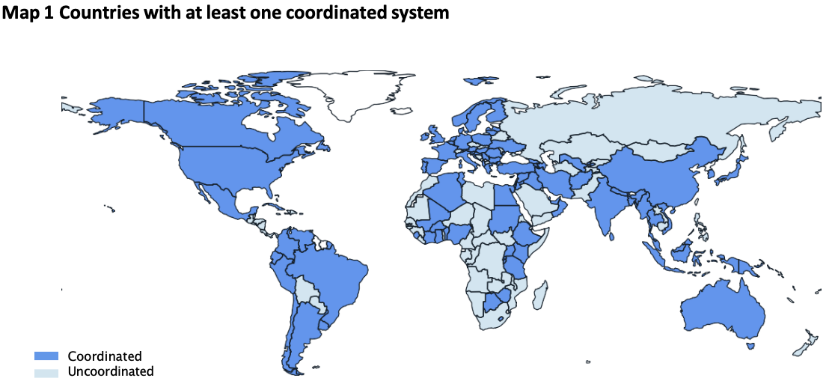
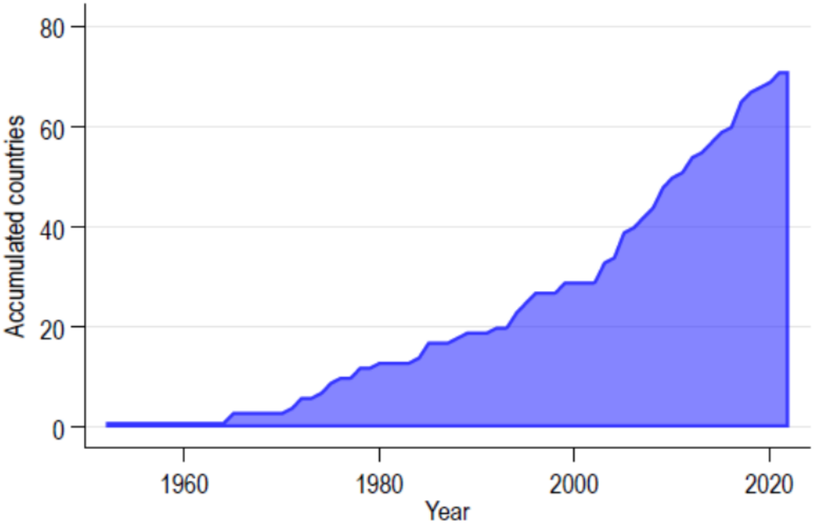


Figure: Source: Neilson (2024)

Recent evolution

Figure 2 Number of accumulated countries with a CCAS



The choice of the mechanism

- What mechanism should we use?
- $\neq$ mechanisms $\Rightarrow$ $\neq$ properties
- Student-optimal **DA** is one of the most popular mechanisms.
- It is the **only stable and strategy-proof** mechanism.
  - *Stability:* a student prefers a school over her assignment $\Rightarrow$ all students assigned to it have higher priority.
  - *Strategy-proofness:* A student cannot do better than submitting truthfully.

The use of different mechanisms

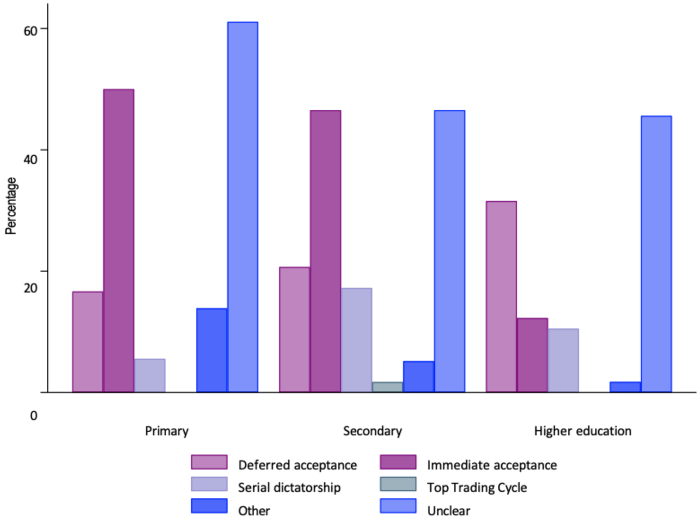


Figure: Source: Neilson (2024)

How does student-proposing DA work?

Preferences and Priorities (unit capacities):

P_1	P_2	P_3
s_2	s_1	s_1
s_1	s_2	s_2
s_3	s_3	s_3

$\succ_1$	$\succ_2$	$\succ_3$
1	2	2
3	1	1
2	3	3

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1	2	2
3	1	1
2	3	3

- **First Step**: Student 1 makes a proposal to school s_2, Student 2 makes a proposal to school s_1, and Student 3 makes a proposal to school s_1. School s_2 accepts Student 1's offer, and school s_1 accepts Student 3's offer because $3 \succ_{s_1} 2$. Student 2 is left alone.

$$\mu_{1-Step} = \begin{pmatrix} 1 & 2 & 3 \\ s_2 & \emptyset & s_1 \end{pmatrix}$$

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$$\mu_{1-Step} = \begin{pmatrix} 1 & 2 & 3 \\ \mathrm{s}_2 & \emptyset & \mathrm{s}_1 \end{pmatrix}$$

- **Second Step**: Student 2 makes a proposal to school s_2. School s_2 accepts Student 2's offer because $2 \succ_{s_2} 1$. Student 1 is left alone.

How does student-proposing DA work?

Preferences and Priorities (unit capacities):

P_1	P_2	P_3
s_2	s_1	s_1
s_1	s_2	s_2
s_3	s_3	s_3

$\succ_1$	$\succ_2$	$\succ_3$
1	2	2
3	1	1
2	3	3

$$\mu_{2-Step} = \begin{pmatrix} 1 & 2 & 3 \\ \emptyset & s_2 & s_1 \end{pmatrix}$$

How does student-proposing DA work?

Preferences and Priorities (unit capacities):

P_1	P_2	P_3
s_2	s_1	s_1
s_1	s_2	s_2
s_3	s_3	s_3

$\succ_1$	$\succ_2$	$\succ_3$
1	2	2
3	1	1
2	3	3

$$\mu_{2-Step} = \begin{pmatrix} 1 & 2 & 3 \\ \emptyset & s_2 & s_1 \end{pmatrix}$$

- **Third Step**: Student 1 makes a proposal to school s_1. School s_1 accepts Student 1's offer because $1 \succ_{s_1} 3$. Student 3 is left alone.

How does student-proposing DA work?

Preferences and Priorities (unit capacities):

P_1	P_2	P_3
s_2	s_1	s_1
s_1	s_2	s_2
s_3	s_3	s_3

$\succ_1$	$\succ_2$	$\succ_3$
1	2	2
3	1	1
2	3	3

$$\mu_{3-Step} = \begin{pmatrix} 1 & 2 & 3 \\ s_1 & s_2 & \emptyset \end{pmatrix}$$

How does student-proposing DA work?

Preferences and Priorities (unit capacities):

P_1	P_2	P_3
s_2	s_1	s_1
s_1	s_2	s_2
s_3	s_3	s_3

$\succ_1$	$\succ_2$	$\succ_3$
1	2	2
3	1	1
2	3	3

$$\mu_{3-Step} = \begin{pmatrix} 1 & 2 & 3 \\ s_1 & s_2 & \emptyset \end{pmatrix}$$

- **Fourth Step**: Student 3 makes a proposal to school s_3. School s_3 accepts Student 3's offer.

$$\mu^{DA} = \begin{pmatrix} 1 & 2 & 3 \\ s_1 & s_2 & s_3 \end{pmatrix}$$

How does student-proposing DA work?

Preferences and Priorities (unit capacities):

P_1	P_2	P_3
s_2	s_1	s_1
s_1	s_2	s_2
s_3	s_3	s_3

$\succ_1$	$\succ_2$	$\succ_3$
1	2	2
3	1	1
2	3	3

$$\mu_{3-Step} = \begin{pmatrix} 1 & 2 & 3 \\ s_1 & s_2 & \emptyset \end{pmatrix}$$

- **Fourth Step**: Student 3 makes a proposal to school s_3. School s_3 accepts student's 3 offers.

$$\mu^{DA} = \begin{pmatrix} 1 & 2 & 3 \\ s_1 & s_2 & s_3 \end{pmatrix}$$

Motivation

- However, it has a drawback, it is **bossy**:

A change in a student's preference can modify the assignment of others without changing her own.

Why is bossiness important?

Preferences and Priorities (unit capacities):

P_1	P_2	P_3
s_2	s_1	s_3
s_1	s_3	s_1

$\succ_1$	$\succ_2$	$\succ_3$
3		2
1		3
2		

Suppose preferences of 1 change to:

P'_1	P_2	P_3
s_1	s_1	s_3
s_2	s_3	s_1

$\succ_1$	$\succ_2$	$\succ_3$
3		2
1		3
2		

Note that the outcome of DA is inefficient. In general, when DA is not efficient, there is a set of "bossy" students.

Why is non-bossiness important? (cont'd)

Suppose that students have preferences over matchings:

$$\begin{pmatrix} 1 & 2 & 3 \\ s_2 & s_3 & s_1 \end{pmatrix} \succ_1 \begin{pmatrix} 1 & 2 & 3 \\ s_2 & s_1 & s_3 \end{pmatrix}$$

Student 1 can manipulate and improve her situation.

Also, it allows for *coalition manipulations.*

What do we do?

- We take a closer look **at** bossiness: What is its scope?
- **New** incentive property: a mechanism is *locally non-bossy* if

whenever a student changes her preferences without changing her assignment, her classmates remain the same.

Equivalently,

a student cannot change her classmates without changing the school to which she is assigned.

- This limits bossiness even in the one-to-one case.

Contribution

1. We first show that DA is locally non-bossy.
2. For any mechanism:

   (Papai) Strategy-proof + non-bossy $\Rightarrow$ Group SP.

   Strategy-proof + locally non-bossy $\Rightarrow$ Locally group SP.
3. Characterize DA without priorities:

IR weak non-wasteful population-monotonic SP weak WrARP weak local non-bossy	$\Longleftrightarrow$	DA for some profile of priorities.

Contribution (cont'd)

4. Introduce "externalities": preferences over matchings.

    - there may no exist a stable matching.
    - school-lexicographic preferences $\Rightarrow \exists$ stable matching but ...
    - it may not exist a stable and SP mechanism.
    - We define school-lexicographic preferences over colleagues:

      students care are first about the school and then only about their classmates,

    - "DA" is stable and SP.
    - Why might a student want to misreport her preferences?
        - Get a better school (SP)
        - Get preferred classmates (local non-bossiness)
    - There is limited room to expand the domain.

Literature

1. **Bossiness of DA.** Many papers: Papái (2000), Ergin (2002)... Afacan and Dur (2017) school-proposing DA is non-bossy for the students. *Our contribution: bossiness of DA is limited*.

2. **Axiomatization of DA.** Kojima and Manea (2010), Morrill (2013), and Ehlers and Klaus (2014, 2016) characterize DA without appealing to stability. *Our contribution: extend Ehlers and Klaus (2016) from unit to multiple capacities.*

3. **Matching with externalities.** Dutta and Massó (1997): lexicographic preferences. Duque and Torres-Martínez (2023) show that a stable and SP mechanism may not exist. *Our contribution: new preference domain for SP and stability.*

Model

- Let N be a set of students and S a set of schools.
- For each school s, $\succ_s$ priority, and capacity $q_s \geq 1$.
- For each student i, preferences P_i defined on $S \cup \{s_0\}$.
- Matching is $\mu : N \to S \cup \{s_0\}$ that respects capacities.
- $\mathcal{M}$ is set of matchings.
- Preference domain $\mathcal{P} = \mathcal{L}^{|N|}$, $\mathcal{L}$ set of strict linear orders.
- Mechanism $\Phi : \mathcal{P} \to \mathcal{M}$. Notation: $\Phi_i(P_i, P_{-i})$ and $\Phi_s(P)$.

Properties

1. μ is **individually rational** if **no** student i prefers s_0 to $\mu(i)$.
2. μ is **stable** if it is IR and there is **no** $(i, s) \in N \times S$ such that:
   - $sP_i\mu(i)$ and either
   - $|\mu(s)| < q_s$ or
   - $i \succ_s j$ for some $j \in \mu(s)$.
3. Φ is **strategy-proof** if there are **no** $i \in N$, $P \in \mathcal{P}$, and $P_i' \in \mathcal{L}$ such that

$$\Phi_i(P_i', P_{-i})P_i\Phi_i(P).$$

Non-bossy and its local version

- Φ is **non-bossy** if for all $i \in N$, $P \in \mathcal{P}$, and $P'_i \in \mathcal{L}$,

$$\Phi_i(P) = \Phi_i(P'_i, P_{-i}) \text{ implies that } \Phi(P) = \Phi(P'_i, P_{-i}).$$

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- **[NEW]** A mechanism Φ is **locally non-bossy** if for all $i \in N$, $P \in \mathcal{P}$, $P'_i \in \mathcal{L}$, and $s \in S \cup \{s_0\}$,

$$\Phi_i(P) = \Phi_i(P'_i, P_{-i}) = s \text{ implies that } \Phi_s(P) = \Phi_s(P'_i, P_{-i}).$$

Group SP and its local version

- Φ is **group strategy-proof** if there are **no** $P \in \mathcal{P}$, $C \subseteq N$, and $P'_C \in \mathcal{L}^{|C|}$ such that:
  - For some $i \in C$, $\Phi_i(P'_C, P_{-C}) \, P_i \, \Phi_i(P)$.
  - For each $j \in C$, $\Phi_j(P'_C, P_{-C}) \, R_j \, \Phi_j(P)$.

- A mechanism Φ is **locally group strategy-proof** if there are **no** $s \in S \cup \{s_0\}$, $P \in \mathcal{P}$, $C \subseteq \Phi_s(P)$, and $P'_C \in \mathcal{L}^{|C|}$ such that:
  - For some $i \in C$, $\Phi_i(P'_C, P_{-C}) \, P_i \, \Phi_i(P)$.
  - For each $j \in C$, $\Phi_j(P'_C, P_{-C}) \, R_j \, \Phi_j(P)$.

Results I

Theorem

The student-optimal DA is locally non-bossy.

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The student-optimal DA is locally non-bossy.

Under DA no student can modify her preferences to change her classmates without changing her school.

But if multiple students, all assigned to the same school, do it?

This is: **locally group non-bossy**.

Proposition

If $\Phi : \mathcal{P} \to \mathcal{M}$ is and a locally non-bossy and strategy-proof mechanism, then it is locally group non-bossy.

Results II: relation with Group SP

Papái (2000):

$$\text{SP} + \text{Non-bossiness} \iff \text{Group SP}$$

In particular, DA is not Group SP.

We show:

$$\text{SP} + \text{Locally Non-bossiness} \Rightarrow \text{Locally Group SP}$$

In particular, DA is Locally Group SP.

Results III: Characterization without priorities

Mechanism: $\Phi : \mathcal{N} \times \mathcal{P} \to \cup_{N \in \mathcal{N}} \mathcal{M}(N)$.

- Φ is **weakly non-wasteful** if $s\, P_i\, \Phi_i(N,P)$ and $\Phi_i(N,P) = s_0$, then $|\Phi_s(N,P)| = q_s$.
- Φ is **population-monotonic** if $N \subseteq N'$, $i \in N$, and $P \in \mathcal{P}$ we have that $\Phi_i(N,P) R_i \Phi_i(N',P)$.
- A mechanism Φ is **weakly WrARP** when for all $N, N' \in \mathcal{N}$, $P \in \mathcal{P}$, and $s \in S$ such that $|N| = |N'| = q_s + 1$ and s is the only acceptable school for every $k \in N \cup N'$,

$$\left[i,j \in N \cap N', i \in \Phi_s(N,P), j \in \Phi_s(N',P) \setminus \Phi_s(N,P) \right]$$

$$\Longrightarrow i \in \Phi_s(N',P).$$

Results II: characterization (cont'd)

- A mechanism Φ is **weakly locally non-bossy** if for all $N \in \mathcal{N}$, $i \in N$, $P \in \mathcal{P}$, $P'_i \in \mathcal{L}$, and $s \in S$, we have that:

$$\Phi_i(N, P) = \Phi_i(N, (P'_i, P_{-i})) = s \text{ implies that}$$
$$\Phi_s(N, P) = \Phi_s(N, (P'_i, P_{-i}))$$

Weakly WrARP and weakly locally non-bossy hold trivially when $q_s = 1 \forall s$.

Results II: characterization (cont'd)

1. Ehlers and Klaus (2016), $\mathbf{q_s} = \mathbf{1}$, $\forall s$:

$$\begin{array}{c} \text{IR} \\ \text{weak non-wasteful} \\ \text{population-monotonicity} \\ \text{SP} \end{array} \quad \Longleftrightarrow \quad \begin{array}{c} \text{DA for some} \\ \text{profile of priorities.} \end{array}$$

2. Our result for general capacities:

$$\begin{array}{c} \text{IR} \\ \text{weak non-wasteful} \\ \text{population-monotonicity} \\ \text{SP} \\ \textbf{weak WrARP} \\ \textbf{weak local non-bossy} \end{array} \quad \Longleftrightarrow \quad \begin{array}{c} \text{DA for some} \\ \text{profile of priorities.} \end{array}$$

Results III: Externalities

Most of the lit. $\Rightarrow$ students care only about the assigned school

Preferences over matchings $\Rightarrow$ many results break down.

Example:(Echenique and Yenmez, 2007) $q_1 = q_2 = 2$

P_1	P_2	P_3
$s_1, \{1,2\}$	$s_2, \{2,3\}$	$s_1, \{1,3\}$
$s_1, \{1,3\}$	$s_1, \{1,2\}$	$s_2, \{2,3\}$
$s_1, \{1\}$	$s_1, \{2\}$	$s_2, \{3\}$
	$s_2, \{2\}$	

$\succ_1$	$\succ_2$
1	3
2	2
3	

IR Matchings:

$$\begin{pmatrix} s_1 & s_2 \\ \{1,2\} & \{3\} \end{pmatrix}, \begin{pmatrix} s_1 & s_2 \\ \{1,3\} & \{2\} \end{pmatrix}, \begin{pmatrix} s_1 & s_2 \\ \{1\} & \{2,3\} \end{pmatrix}$$

$\nexists$ **stable matching**

Results III: externalities (cont'd)

- **School-lexicographic preference** $(\mathcal{D})$: $\trianglerighteq_i$ defined on $\mathcal{M}$ such that, for any $\mu, \eta \in \mathcal{M}$:
  - If $\mu(i) \neq \eta(i)$, then either $\mu \vartriangleright_i \eta$ or $\eta \vartriangleright_i \mu$, where $\vartriangleright_i$ is the strict part of $\trianglerighteq_i$.
  - If $\mu \vartriangleright_i \eta$ and $\mu(i) \neq \eta(i)$, then $\mu' \vartriangleright_i \eta'$ for all matchings $\mu', \eta' \in \mathcal{M}$ such that $\mu'(i) = \mu(i)$ and $\eta'(i) = \eta(i)$.
- In $\mathcal{D}$ we can define $P(\trianglerighteq) = (P_i(\trianglerighteq))_{i \in N} \in \mathcal{P}$.
- Recover stability but a stable and SP mechanism may not exist (Duque and Torres-Martínez, 2023).

Results III: externalities (cont'd)

We further restrict the domain.

School-lexicographic preference over colleagues: $\mathcal{D}_\mathrm{c} \subseteq \mathcal{D}$ is the set of profiles $(\trianglerighteq_i)_{i \in N}$ such that $\mu \triangleright_i \eta$ and $\mu(i) = \eta(i) = s$ imply that $\mu(s) \neq \eta(s)$.

In $\mathcal{D}_\mathrm{c}$ a student is indifferent between two matchings where she is assigned to same school with the same classmates.

Theorem

In any school choice context $(S, N, \succ, q)$, the mechanism $\overline{\mathrm{DA}} : \mathcal{D}_\mathrm{c} \to \mathcal{M}$ defined by $\overline{\mathrm{DA}}(\trianglerighteq) = \mathrm{DA}(P(\trianglerighteq))$ is stable and strategy-proof.

Moreover, is the only stable and strategy-proof mechanism.

Limited room to expand the domain

$q_1 = 2, q_2 = q_3 = q_4 = 1$. All but 1 have preferences in $\mathcal{D}_c$.

$P_1(\trianglerighteq)$	$P_2(\trianglerighteq)$	$P_3(\trianglerighteq)$	$P_4(\trianglerighteq)$	$P_5(\trianglerighteq)$
s_3	s_2	s_1	s_1	s_4
	s_1	s_2		

$\succ_1$	$\succ_2$	$\succ_3$	$\succ_4$
4	3	1	2
2	2	2	5
1			
3			
5			

Notice that $[N, S, \succ, q, P(\trianglerighteq)]$ has only two stable matchings:

$$\begin{aligned} \mu &= ((1, s_3), (2, s_1), (3, s_2), (4, s_1), (5, s_4)), \text{ (school-optimal)} \\ \eta &= ((1, s_3), (2, s_2), (3, s_1), (4, s_1), (5, s_4)) \text{ (student-optimal)}. \end{aligned}$$

Suppose $\mu \trianglerighteq_1 \eta$.
If $\Phi(\trianglerighteq) = \mu$, consider $P_2' : s_2, s_4$, and η is the only SM.
If $\Phi(\trianglerighteq) = \eta$, consider $P_1' : s_1, s_3, \ldots$, and μ is the only SM.

Concluding Remarks

- DA is locally non-bossy: a student cannot change her classmates without changing her own school.
- For any SP mechanism, local non-bossiness guarantees that no coalition of students **assigned to the same school** can misrepresent their preferences to either:
  - improve their assignments or
  - maintain their school while modifying their classmates.
- DA still performs well when students care about the assignments of others, as long as they first consider their assigned school and then their classmates.
- The incompatibility between stability and SP in contexts where students prioritize their own school is caused by the fact that preferences extend beyond their classmates.

Thanks!