THE STRONG EFFECTS OF WEAK EXTERNALITIES ON SCHOOL CHOICE

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ABSTRACT. In classical school choice contexts there exists a centralized assignment procedure that is stable and strategy-proof: the Gale-Shapley student-optimal stable mechanism. We show that this property is not satisfied when externalities are incorporated into the model, even in scenarios in which students are primarily concerned about their own placement (weak externalities). Indeed, although weak externalities have no effects on stability, there are school choice contexts in which no stable and strategy-proof mechanism exists. Furthermore, we show that stability and strategy-proofness are compatible if and only if schools' priorities are Ergin-acyclic. This strong effect of weak externalities on incentives is related to the incompatibility between stability, strategy-proofness, and non-bossiness in classical school choice problems.

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1. INTRODUCTION

Since the seminal works of Balinski and Sönmez (1999) and Abdulkadiroğlu and Sönmez (2003) on college admissions and school choice, there has been extensive research on centralized admission systems. In classical school choice problems, the literature has focused on the analysis of student behavior and the characterization of fairness and efficiency properties of the distribution of students into schools (cf., Pathak (2011, 2017), Abdulkadiroğlu (2013), and Kojima (2017)). In this way, and to prevent families from having to use complex strategies to participate in the admission processes, special attention has been paid to *strategy-proof mechanisms*—assignment procedures in which students have incentives to truthfully report their preferences. However, strategy-proof mechanisms generate tensions between stability and Pareto efficiency.[1] The *student-optimal stable mechanism* (SOSM) is strategy-proof and stable, but it is not Pareto efficient; alternatively, the *top trading cycles mechanism* (TTC) is strategy-proof and Pareto efficient, but it is not stable.[2] Moreover, strategy-proofness, stability, and Pareto efficiency are incompatible (cf., Alcalde and Barberà (1994)). Therefore, the choice of a strategy-proof mechanism often depends on the importance that policymakers give to stability and efficiency.

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[1] *Stability* requires that the matching between schools and students be *fair*, *non-wasteful*, and *individually rational* (cf., Balinski and Sönmez (1999)). Fairness ensures that no one wants to claim a seat at a school arguing that it was assigned to a lower-priority student; non-wastefulness requires that no student wants a seat that was not assigned; individual rationality guarantees that each student is matched with a school that she considers admissible.

[2] The SOSM mechanism associates with each preference profile the matching obtained by the deferred-acceptance algorithm when students make proposals. For more details about the properties of this mechanism, see the works of Gale and Shapley (1962), Dubins and Freedman (1981), Roth (1982), and Ergin (2002).

In this direction, many centralized admission systems have privileged stability over efficiency. Among other places, Boston, Chicago, New York, Paris, Chile, Finland, Ghana, Romania, and Turkey assign students to public school seats through the SOSM mechanism.[3] To justify this choice it can be argued that SOSM is weakly Pareto efficient and implements the best stable outcome for students (see Gale and Shapley (1962), Gale and Sotomayor (1985), Balinski and Sönmez (1999)).[4] Since schools' priorities have an intuitive role in the final assignment of SOSM, it could also be argued that this mechanism is more transparent than TTC (cf., Leshno and Lo (2021)).

Unfortunately, classical school choice problems ignore the existence of externalities: students do not care about the distribution of schools' vacancies among the rest of the candidates (cf., Abdulkadiroğlu and Sönmez (2003)). However, the situation of others can have relevant effects on the well-being of some students. For instance, students' educational achievements may increase with the quality of the schools attended by children in their social network (cf., Calvó-Armengol, Patacchini, and Zenou (2009)).

The presence of externalities can compromise the existence of a stable matching. Thus, restrictions on students' preferences or schools' priorities can be required to recover the solvability of the school choice problem (see Dutta and Massó (1997), Echenique and Yenmez (2007), Bodine-Baron et al. (2011), Bykhovskaya (2020), Pycia and Yenmez (2022)). Furthermore, even in contexts in which externalities have no effects on stability, their presence could change students' incentives to reveal information. In particular, some popular mechanisms that satisfy desirable properties in traditional environments could cease to work well under externalities.

In this paper, we want to improve our understanding of the effects of externalities on student behavior in centralized assignment procedures. Our objective is to show that even *weak externalities* have a deep effect on students' incentives to reveal information about their preferences. We assume that students have preferences defined on the set of matchings, but they are primarily concerned about their own placement (*egocentric preferences*). Each school has a strict priority ordering of all students and a maximum capacity, but there are enough vacancies in the system to allocate everyone. Although the rules determining the *school choice context*—which is characterized by schools' priorities and quotas—are publicly known, students' preferences are not observable.

Notice that weak externalities have no effects on stability. Indeed, underlying any egocentric preference relation there is a *standard preference relation*—a linear order defined on the set of schools. Thus, the set of stable matchings under students' egocentric preferences is non-empty, because it coincides with the set of stable matchings of the problem without externalities in which students have the underlying standard preferences (see Roth and Sotomayor (1989)).[5]

In our first result about the effects of weak externalities on students' incentives, we show that there are school choice contexts in which no stable mechanism is strategy-proof (see Theorem 1). This impossibility

The TTC mechanism associates with each preference profile the matching obtained by the top trading cycles algorithm proposed by Abdulkadiroğlu and Sönmez (2003). For additional details about the properties of this mechanism, see Pápai (2000) and Abdulkadiroğlu and Sönmez (2003).

[3]For detailed descriptions of these admission systems, see the works of Pathak and Sönmez (2013), Pop-Eleches and Urquiola (2013), Abdulkadiroğlu, Angrist, and Pathak (2014), Hiller and Tercieux (2014), Salonen (2014), Akyol and Krishna (2017), Ajayi (2022), and Correa et al. (2022).

[4]Some of the centralized admissions systems cited above restrict the number of eligible schools that students can declare. As a consequence, SOSM is no longer strategy-proof (see Haeringer and Klijn (2009)). However, SOSM is the least manipulable of the mechanisms that implement matchings that are stable with respect to reported truncated preferences (see Pathak and Sönmez (2013, Lemma 1)).

[5]For marriage markets, the innocuousness of weak externalities on stability was pointed out by Sasaki and Toda (1996) and Fonseca-Mairena and Triossi (2023).

holds in any domain containing all egocentric preference profiles and it is still valid when students consider all schools admissible or they can declare up to a fixed number of them to be acceptable (see Remark 1).

Evidently, these properties contrast with what occurs in classical school choice problems, in which the SOSM mechanism is strategy-proof for every school choice context (see Dubins and Freedman (1981), Roth (1982)). The intuition behind our results is straightforward: with weak externalities a student may misreport preferences to either change her school to a preferred one or maintain her placement and change the distribution of others, improving her situation as a consequence of second-order factors captured by weak externalities. Although the first reason for misreporting preferences is already present in classical school choice problems, the second one emerges in the presence of weak externalities. Avoiding the first incentive to lie is related to ensuring strategy-proofness in classical school choice problems, while avoiding the second incentive to lie is related to guaranteeing *non-bossiness* in classical school choice problems (i.e., the impossibility to change the situation of others without altering the own placement). In other words, our impossibility result is related to the fact that—in the absence of externalities—no stable mechanism is strategy-proof and non-bossy for all school choice contexts.[6]

To formalize the relation between strategy-proofness under weak externalities and the combination of strategy-proofness and non-bossiness in classical school choice contexts, we consider the class of *myopic mechanisms*: assignment procedures that apply a mechanism for classical school choice problems to the standard preferences underlying students' egocentric preferences. Given a school choice context, we show that a mechanism for problems without externalities is both strategy-proof and non-bossy if and only if the induced myopic mechanism is strategy-proof (see Theorem 2). As a consequence, the incompatibility between strategy-proofness, stability, and non-bossiness in classical school choice contexts ensures that *within* the class of myopic mechanisms no one is stable and strategy-proof.

It follows from our results that Pareto efficiency dominates stability under weak externalities. Indeed, as a consequence of our Theorem 2, the myopic mechanism induced by TTC is strategy-proof, Pareto efficient, and individually rational.[7] Therefore, in the search for a mechanism that is strategy-proof and individually rational for *every* school choice context, it follows from Theorem 1 that there is no other alternative than to favor Pareto efficiency over stability.

To assess the depth of the incompatibility between stability and strategy-proofness under weak externalities, we also study what constraints on schools' priorities guarantee that a stable and strategy-proof mechanism exists. Notice that, in a context without externalities, the absence of Ergin-cycles in schools' priorities is necessary and sufficient to ensure that the SOSM mechanism is strategy-proof and non-bossy (see Ergin (2002), Narita (2021)).[8] Hence, it follows from our Theorem 2 that the myopic mechanism induced by SOSM is strategy-proof if and only if schools' priorities are Ergin-acyclic (see Proposition 1). Also, the proof of Alcalde and Barberà (1994, Theorem 3) can be adapted to guarantee the following property: if there is a stable and strategy-proof mechanism defined in the domain of egocentric preferences, then it coincides with the myopic mechanism induced by SOSM

[6]Given a school choice context, assume that each student has a strict rank for schools and may declare some of them inadmissible. In this context, Alcalde and Barberà (1994, Theorem 3) show that the SOSM mechanism is the only one that is stable and strategy-proof in the whole domain of students' preferences. However, as pointed out by Roth (1982, Section 6), this mechanism is bossy.

[7]Notice that, in classical school choice contexts, the TTC mechanism is strategy-proof, non-bossy, Pareto efficient, and individually rational (see Pápai (2000), Abdulkadiroğlu and Sönmez (2003)). Moreover, the set of Pareto efficient matchings for a profile of egocentric preferences includes the matchings that are Pareto efficient for the underlying standard preferences.

[8]Paraphrasing Ergin's words, "a priority structure is *acyclical* if it never gives rise to situations where a student can block a potential settlement between any other two students without affecting his own position".

(see Proposition 2). Therefore, we conclude that there exists a stable and strategy-proof mechanism under weak externalities if and only if schools' priorities are Ergin-acyclic (see Theorem 3).

When priorities are school-specific—as in many centralized admission systems—it is difficult to ensure the absence of Ergin-cycles. Thus, the result of Theorem 3 reinforce the idea that weak externalities have profound effects on incentives.

The rest of the paper is organized as follows. Section 2 describes the characteristics of a school choice problem with weak externalities. Section 3 shows the existence of school choice contexts in which no stable mechanism is strategy-proof. Section 4 formalizes the relationship between strategy-proofness under weak externalities and non-bossiness without externalities. Section 5 determines necessary and sufficient conditions for schools' priorities to guarantee that stability and strategy-proofness are compatible. In Section 6 some concluding remarks are provided.

2. The model

In a *school choice problem with weak externalities* $(S, H, \succ, q, R)$ there is a set S of schools and a set H of students, where $|S| \geq 2$ and $|H| \geq 3$. School s has a quota $q_s \geq 1$ and ranks students through a linear order $\succ_s$ defined on H.[9] Although each school may have a limited capacity, there are enough vacancies to accommodate all students, as $|H| \leq \sum_{s\in S} q_s$. We refer to $\succ \equiv (\succ_s)_{s\in S}$ as the *priority structure* and to $q \equiv (q_s)_{s\in S}$ as the *vector of quotas*.

A distribution of schools' seats among students, or *matching*, is a function $\mu : H \to S \cup \{\otimes\}$ such that $|\mu^{-1}(s)| \leq q_s$ for all $s \in S$, where $\mu^{-1}(s) = \{h \in H : \mu(h) = s\}$. Hence, a matching μ enrolls the set of students $\mu^{-1}(s)$ in school $s \in S$ and assigns a seat in $\mu(h) \in S \cup \{\otimes\}$ to student h. Being assigned to $\otimes$ is interpreted as not being assigned to any school.[10] Let $\mathcal{M}$ be the set of matchings.

Students' preferences exhibit *weak externalities*. Hence, each $h \in H$ has a complete and transitive preference relation R_h defined on $\mathcal{M}$, but she is primarily concerned about her own school. More formally, we assume that R_h is *egocentric* in the following sense:

(i) If h is indifferent between matchings μ and η, then $\mu(h) = \eta(h)$.

(ii) If h strictly prefers matching μ to matching η and $\mu(h) \neq \eta(h)$, then $\mu' P_h \eta'$ for all $\mu', \eta' \in \mathcal{M}$ such that $\mu'(h) = \mu(h)$ and $\eta'(h) = \eta(h)$, where P_h denotes the strict part of R_h.

Although the rules determining the *school choice context* $(S, H, \succ, q)$ are publicly known, the profile $R \equiv (R_h)_{h\in H}$ is not observable. We denote by $\mathcal{R}^{\text{ego}}$ the domain of students' preference profiles $(R_h)_{h\in H}$ in which every R_h is a complete, transitive, and egocentric preference relation defined on $\mathcal{M}$.[11]

Let $\mathcal{R}^{\text{std}}$ be the set of profiles $(\sigma_h)_{h\in H}$ such that every σ_h is a linear order defined on $S \cup \{\otimes\}$. For each $R = (R_h)_{h\in H} \in \mathcal{R}^{\text{ego}}$, let $(\sigma(R_h))_{h\in H} \in \mathcal{R}^{\text{std}}$ be the linear orders such that $s\,\sigma(R_h)\,s'$ as long as $\mu P_h \mu'$ for every $\mu, \mu' \in \mathcal{M}$ such that $\mu(h) = s$ and $\mu'(h) = s'$. We refer to $\sigma(R_h)$ as the *standard preference associated with* R_h and we denote by $\sigma(R)$ the profile $(\sigma(R_h))_{h\in H}$.

Given $(R_h)_{h\in H} \in \mathcal{R}^{\text{ego}}$, consider the following concepts relating to efficiency and fairness:

- A matching μ is *individually rational* when $\mu(h)\sigma(R_h)\otimes$ for any student h such that $\mu(h) \in S$.
- A matching μ is *Pareto efficient* when there is no matching $\mu' \in \mathcal{M}$ such that $\mu' R_h \mu$ for every $h \in H$ and $\mu' P_{\hat{h}} \mu$ for some $\hat{h} \in H$.

[9] A *linear order* is a complete, transitive, and strict preference relation.

[10] We can also interpret $\otimes$ as an outside option which represents the decision to apply to a private school.

[11] In marriage markets with externalities, Sasaki and Toda (1996) refer to egocentric preferences as *order preserving preferences*. Egocentric preferences were introduced in Shapley-Scarf housing markets by Hong and Park (2018, 2022).

- A matching μ is *weakly Pareto efficient* when there is no $\mu' \in \mathcal{M}$ such that $\mu' P_h \mu$, $\forall h \in H$.
- It is said that a student h *claims an empty seat at school* s when $s\,\sigma(R_h)\,\mu(h)$ and $|\mu^{-1}(s)| < q_s$. The matching μ is *non-wasteful* when no student claims an empty seat at any school.
- It is said that a student h has *justified-envy towards a student* h' enrolled at school $s = \mu(h')$ whenever $s\,\sigma(R_h)\,\mu(h)$, $|\mu^{-1}(s)| = q_s$, and $h \succ_s h'$. The matching μ is *envy-free* when no student has justified-envy towards another one.

A matching is *stable* when it is individually rational, non-wasteful, and envy-free.

Some remarks about the effect of weak externalities on stability:

- Since preferences are *egocentric*, when a student transfers to a better school, her situation improves regardless of the changes that other students may implement later. For this reason, those that claim an empty seat or have justified-envy do not take into account the potential reactions of other students.
- Given a preference profile $R \in \mathcal{R}^{\text{ego}}$, the problem $(S, H, \succ, q, R)$ has the same stable matchings that the school choice problem without externalities $(S, H, \succ, q, \sigma(R))$. Hence, it follows from Roth and Sotomayor (1989, Lemma 1) that any school choice problem with weak externalities has a stable matching.

Therefore, the presence of egocentric preferences has no effect on the solvability of a school choice problem (Sasaki and Toda (1996), Fonseca-Mairena and Triossi (2023)).

3. Stability and strategy-proofness: an impossibility result

In this section, we formalize the idea that weak externalities have a deep effect on students' incentives to reveal information about their preferences. That is, we will show that there are school choice contexts $(S, H, \succ, q)$ where no protocol associating a stable matching with each preference profile $R \in \mathcal{R}^{\text{ego}}$ makes truth-telling a dominant strategy for students.

Given a school choice context $(S, H, \succ, q)$, a *mechanism* or *assignment procedure* $\Gamma : \mathcal{R}^{\text{ego}} \to \mathcal{M}$ is a function that associates a matching with each students' preference profile.

Consider the following properties:

- A mechanism Γ is *stable* when the matching $\Gamma[R]$ is stable in $(S, H, \succ, q, R)$, for all $R \in \mathcal{R}^{\text{ego}}$.
- A mechanism Γ is *(weakly) Pareto efficient* when the matching $\Gamma[R]$ is (weakly) Pareto efficient in the problem $(S, H, \succ, q, R)$, for all $R \in \mathcal{R}^{\text{ego}}$.
- A mechanism Γ is *strategy-proof* when there is no student $\hat{h}$ such that,

$$\Gamma[(R_h)_{h \neq \hat{h}}, R'_{\hat{h}}]\, P_{\hat{h}}\, \Gamma[(R_h)_{h \in H}]$$

  for some preference profiles $(R_h)_{h \in H}$, $(R'_h)_{h \in H} \in \mathcal{R}^{\text{ego}}$.

Hence, Γ is strategy-proof when truth-telling is a dominant strategy in the non-cooperative game in which students report preferences $R \in \mathcal{R}^{\text{ego}}$ and the matching $\Gamma[R]$ is implemented.

In school choice problems without externalities—in which students' preferences are given by linear orders defined on $S \cup \{\otimes\}$—the *student-optimal stable mechanism* $\text{DA}_{(\succ,q)} : \mathcal{R}^{\text{std}} \to \mathcal{M}$ associates with each $\sigma \in \mathcal{R}^{\text{std}}$ the matching resulting from the application of the student-proposing deferred-acceptance algorithm to $(S, H, \succ, q, \sigma)$ (see Gale and Shapley (1962)).

It is well-known that, for every school choice context $(S, H, \succ, q)$, the mechanism $\mathrm{DA}_{(\succ,q)}$ is stable and strategy-proof in $\mathcal{R}^{\mathrm{std}}$ (see Gale and Shapley (1962), Dubins and Freedman (1981), and Roth (1982)). The following result shows that no mechanism with these characteristics exists for school choice problems with weak externalities.

Theorem 1. *There are school choice contexts $(S, H, \succ, q)$ in which no stable and strategy-proof mechanism exists.*

Proof. Assume that $S = \{s_1, \ldots, s_n\}$ and $H = \{h_1, \ldots, h_n\}$. Each school has one seat available ($q_s = 1$, for all $s \in S$) and the priority structure $\succ = (\succ_s)_{s \in S}$ satisfies the following conditions:[12]

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$	$\cdots$	$\succ_{s_{n-1}}$	$\succ_{s_n}$
h_2	h_1	h_3	h_4	$\cdots$	h_{n-1}	h_2
h_3	h_2	$\vdots$	$\vdots$	$\vdots$	$\vdots$	h_n
h_1	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Let $R = (R_h)_{h \in H} \in \mathcal{R}^{\mathrm{ego}}$ be a preference profile such that the standard preferences associated with it satisfy the following properties:

$\sigma(R_{h_1})$	$\sigma(R_{h_2})$	$\sigma(R_{h_3})$	$\cdots$	$\sigma(R_{h_n})$
s_1	s_2	s_3	$\cdots$	s_n
s_2	s_1	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$

Given the matchings

$$\mu = \begin{pmatrix} h_1 & h_2 & h_3 & \cdots & h_n \\ s_1 & s_2 & s_3 & \cdots & s_n \end{pmatrix} \qquad \text{and} \qquad \mu' = \begin{pmatrix} h_1 & h_2 & h_3 & \cdots & h_n \\ s_2 & s_1 & s_3 & \cdots & s_n \end{pmatrix},$$

assume that the preferences of student h_3 are such that $\mu' P_{h_3} \mu$.

In this context, μ and μ' are the only stable matchings of $(S, H, \succ, q, R)$. Therefore, for any stable mechanism $\Gamma : \mathcal{R}^{\mathrm{ego}} \to \mathcal{M}$ we have that $\Gamma(R) \in \{\mu, \mu'\}$.

Suppose that $\Gamma(R) = \mu$. If $\tilde{R}_{h_3}$ is an egocentric preference such that the associated standard preferences satisfy $s_1\, \sigma(\tilde{R}_{h_3})\, s_3\, \sigma(\tilde{R}_{h_3}) \cdots$, then μ' is the only stable matching when students' preferences are $(R_{-h_3}, \tilde{R}_{h_3})$. Thus, the student h_3 has incentives to misrepresent her preferences, because $\Gamma(R_{-h_3}, \tilde{R}_{h_3}) P_{h_3} \Gamma(R)$.

Suppose that $\Gamma(R) = \mu'$. If $\tilde{R}_{h_2}$ is an egocentric preference such that the associated standard preferences satisfy $s_2\, \sigma(\tilde{R}_{h_2})\, s_n\, \sigma(\tilde{R}_{h_2}) \cdots$, then μ is the only stable matching when students' preferences are $(R_{-h_2}, \tilde{R}_{h_2})$. Thus, the student h_2 has incentives to misrepresent her preferences, because $\Gamma(R_{-h_2}, \tilde{R}_{h_2}) P_{h_2} \Gamma(R)$.

Therefore, in any school choice problem in which $(S, H, \succ, q)$ complies with the requirements above, no stable mechanism $\Gamma : \mathcal{R}^{\mathrm{ego}} \to \mathcal{M}$ is strategy-proof. $\square$

[12] In the description of priorities or preferences, the vertical dots stand for arbitrary ordering of students or schools.

Evidently, it follows from Theorem 1 that stability and strategy-proofness are incompatible in any preference domain containing $\mathcal{R}^{\text{ego}}$. Moreover, the non-existence of stable and strategy-proof mechanisms holds even when the outside option of declaring some schools inadmissible is eliminated. Indeed, the proof of Theorem 1 assumes that all schools are acceptable for all students.

Any problem $(S, H, \succ, q, R)$ obeying the restrictions described in the proof of Theorem 1 has only two stable outcomes, which in turn coincide with the *student-optimal* and the *school-optimal* stable matchings of the induced problem without externalities $(S, H, \succ, q, \sigma(R))$.[13] Since students' preferences are egocentric, the same arguments of Gale and Sotomayor (1985, Theorem 1) can be applied to show that at least one student has incentives to misreport preferences when a stable mechanism implements the *school-optimal* matching of $(S, H, \succ, q, \sigma(R))$.

Therefore, from the point of view of students' strategic behavior, the main difference between the model without externalities and our framework is that a student may have incentives to misreport preferences even when the *student-optimal* stable matching of $(S, H, \succ, q, \sigma(R))$ is implemented. Indeed, the presence of externalities may give incentives to a student to misreport preferences in order to change the placement of others, provided that it can be done without affecting her situation. This is what happens in any of the school choice problems described in the proof of Theorem 1.

Intuitively, strategy-proofness under weak externalities is related to strategy-proofness and non-bossiness in classical school choice problems. This relationship will be formalized in Theorem 2.

Given a problem $(S, H, \succ, q, R)$, it is said that a school s is *attainable* for student h if there exists a stable matching μ such that $\mu(h) = s$. Since students' preferences are egocentric and the sets of stable matchings of $(S, H, \succ, q, R)$ and $(S, H, \succ, q, \sigma(R))$ coincide, it follows from Gale and Shapley (1962, Theorem 2) that $(S, H, \succ, q, R)$ has a stable outcome in which every student gets a seat at her preferred attainable school.

However, despite what happens without externalities, under weak externalities there are school choice problems with no *student-optimal* stable matching. Indeed, in any of the school choice problems described in the proof of Theorem 1, the students do not agree on which of the two stable matchings is the best.

Remark 1 (*Incentives under weak externalities in constrained school choice*)

A common practice in some real-life school choice systems consists of asking students to submit *truncated preferences*, in order to limit the number of schools that can be reported. For instance, in the centralized systems of Chicago, Singapore, and Ghana students can declare up to six alternatives (see Pathak and Sönmez (2013), Teo, Sethuraman, and Tan (2001), Ajayi (2022)).[14]

When this restriction is implemented in scenarios without externalities, the mechanism $\text{DA}_{(\succ,q)}$ ceases to be strategy-proof for all $(S, H, \succ, q)$ (see Haeringer and Klijn (2009)). We complement this result: under weak externalities, if students can report at least two admissible schools in their egocentric preferences, there are school choice contexts in which *no* stable mechanism is strategy-proof. Indeed, only the two best alternatives of each student are required to prove the Theorem 1. □

[13] A stable matching is *student-optimal* when it is weakly preferred by every student to any other stable outcome. Using schools' priorities to order students, the *school-optimal* stable matching is defined in an analogous way.

[14] For additional examples, see Agarwal and Somaini (2018, Table 1) and Fack, Grenet, and He (2019, Table 1).

4. Myopic mechanisms

In this section, we relate the incentives to reveal information in our framework with those in scenarios where students' preferences are defined over $S \cup \{\otimes\}$ instead of $\mathcal{M}$, referred to as *classical school choice problems*. Our findings will be crucial to determining restrictions on preference domains that ensure the existence of a stable mechanism that is strategy-proof under weak externalities.

Given a school choice context $(S, H, \succ, q)$, in the absence of externalities a *mechanism* or *assignment procedure* is a function $\Phi : \mathcal{R}^{\text{std}} \to \mathcal{M}$ that associates a matching with every profile of linear orders $(\sigma_h)_{h \in H}$ defined on $S \cup \{\otimes\}$. For each mechanism $\Phi : \mathcal{R}^{\text{std}} \to \mathcal{M}$, consider the following properties:

- Φ is *stable* when the matching $\Phi[\sigma]$ is stable in $(S, H, \succ, q, \sigma)$, for all $\sigma = (\sigma_h)_{h \in H} \in \mathcal{R}^{\text{std}}$.
- Φ is *(weakly) Pareto efficient* when $\Phi[\sigma]$ is (weakly) Pareto efficient in $(S, H, \succ, q, \sigma)$, for any $\sigma = (\sigma_h)_{h \in H} \in \mathcal{R}^{\text{std}}$.
- Φ is *strategy-proof* when there is no student h such that $\Phi[\sigma_{-h}, \sigma'_h](h) \, \sigma_h \, \Phi[\sigma](h)$, for some preference profiles $\sigma, \sigma' \in \mathcal{R}^{\text{std}}$.
- Φ is *non-bossy* as long as, for all student $h \in H$ and $\sigma, \sigma' \in \mathcal{R}^{\text{std}}$, $\Phi[\sigma_{-h}, \sigma'_h](h) = \Phi[\sigma](h)$ implies that $\Phi[\sigma_{-h}, \sigma'_h] = \Phi[\sigma]$.

Hence, the mechanism is non-bossy when no student can change the school of someone else without being affected by misreporting her preferences (cf., Satterthwaite and Sonnenschein (1981)).[15]

Under weak externalities, an assignment procedure can be determined by asking students to submit a ranking of schools. More formally, by acting on the standard preferences associated with egocentric preferences, an assignment procedure $\Phi : \mathcal{R}^{\text{std}} \to \mathcal{M}$ for school choice problems without externalities generates a mechanism $\Gamma : \mathcal{R}^{\text{ego}} \to \mathcal{M}$ through the rule $\Gamma[R] = \Phi[\sigma(R)]$. We refer to such a mechanism as the *myopic mechanism* induced by Φ.

Given a school choice context $(S, H, \succ, q)$, it is well-known that the *student-optimal stable mechanism* $\text{DA}_{(\succ,q)} : \mathcal{R}^{\text{std}} \to \mathcal{M}$ is strategy-proof and weakly Pareto efficient. Moreover, the matching $\text{DA}_{(\succ,q)}[\sigma]$ is weakly preferred by every student to any other stable outcome of $(S, H, \succ, q, \sigma)$.[16]

These properties are not inherited by the myopic mechanism $\text{DA}^{\text{ego}}_{(\succ,q)} : \mathcal{R}^{\text{ego}} \to \mathcal{M}$ induced by $\text{DA}_{(\succ,q)}$. Indeed, for some specifications of the school choice context, $\text{DA}^{\text{ego}}_{(\succ,q)}$ is not strategy-proof and it does not always generates a student-optimal stable matching (see Theorem 1). Moreover, as the following example illustrates, the outcome of $\text{DA}^{\text{ego}}_{(\succ,q)}$ is not necessarily weakly Pareto efficient.

Example 1. Let $(S, H, \succ, q, R)$ be a problem such that $S = \{s_1, s_2, s_3\}$, $H = \{h_1, h_2, h_3, h_4\}$, and $q = (q_{s_1}, q_{s_2}, q_{s_3}) = (1, 1, 2)$. Suppose that the priority structure $\succ = (\succ_s)_{s \in S}$ and the *standard preferences* associated with $R = (R_h)_{h \in H}$ are given by

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\sigma(R_{h_1})$	$\sigma(R_{h_2})$	$\sigma(R_{h_3})$	$\sigma(R_{h_4})$
h_3	h_2	h_1	s_1	s_1	s_2	s_3
h_2	h_1	h_2	s_2	s_3	s_1	s_1
h_1	h_3	h_3	s_3	s_2	s_3	s_2
h_4	h_4	h_4	$\otimes$	$\otimes$	$\otimes$	$\otimes$

[15] Throughout the paper, we refer to a mechanism as *bossy* when it is not non-bossy.

[16] See Gale and Shapley (1962), Dubins and Freedman (1981, Theorem 9), Roth (1982, Theorem 5), and Gale and Sotomayor (1985, Theorem 3) for details of the proof of these properties.

Moreover, given the matchings

$$\mu = \begin{pmatrix} h_1 & h_2 & h_3 & h_4 \\ s_2 & s_3 & s_1 & s_3 \end{pmatrix}, \qquad \mu' = \begin{pmatrix} h_1 & h_2 & h_3 & h_4 \\ s_1 & s_3 & s_2 & s_3 \end{pmatrix}.$$

suppose that students h_2 and h_4 strictly prefer μ' to μ.

In this context, $\mathrm{DA}^{\mathrm{ego}}_{(\succ,q)}[R] = \mu$ and all students strictly prefer μ' to μ. Hence, $\mathrm{DA}^{\mathrm{ego}}_{(\succ,q)}$ is not weakly Pareto efficient. □

The problem described in Example 1 allows us to give an alternative proof that there are school choice contexts in which $\mathrm{DA}^{\mathrm{ego}}_{(\succ,q)}$ is not strategy-proof in the domain $\mathcal{R}^{\mathrm{ego}}$ (see Theorem 1):

> If every student $h \neq h_2$ truthfully reports her preferences, then h_2 has incentives to report egocentric preferences R'_{h_2} such that $\sigma(R'_{h_2})$ satisfies $s_3\, \sigma(R'_{h_2})\, s_1\, \sigma(R'_{h_2})\, s_2\, \sigma(R'_{h_2})\, \otimes$. Indeed, $\mathrm{DA}^{\mathrm{ego}}_{(\succ,q)}[(R_{-h_2}, R'_{h_2})] = \mu'$ and h_2 strictly prefers μ' to μ.

This argument also shows that $\mathrm{DA}_{(\succ,q)} : \mathcal{R}^{\mathrm{std}} \to \mathcal{M}$ is *bossy* in some school choice contexts. In fact, although the student h_2 does not change her school when she misreports her preferences, the implemented matching changes (cf., Roth (1982, Section 6)).

Remark 2 (*Non-existence of stable and weakly efficient mechanisms*)

Under weak externalities there are school choice contexts such that no stable mechanism defined on $\mathcal{R}^{\mathrm{ego}}$ is weakly Pareto efficient. Indeed, the problem described in Example 1 has a *unique* stable matching that is not weakly Pareto efficient. □

The following result characterizes, from the point of view of students' incentives to reveal information, the relationship between an assignment procedure for classical school choice problems and the *myopic mechanism* induced by it.

Theorem 2. *Given a school choice context $(S, H, \succ, q)$ and a mechanism $\Phi : \mathcal{R}^{\mathrm{std}} \to \mathcal{M}$, consider the mechanism $\Gamma : \mathcal{R}^{\mathrm{ego}} \to \mathcal{M}$ such that*

$$\Gamma[R] = \Phi[\sigma(R)], \qquad \forall R \in \mathcal{R}^{\mathrm{ego}}.$$

Then, Γ is strategy-proof if and only if Φ is strategy-proof and non-bossy.

Proof. The fact that Φ is strategy-proof and non-bossy as long as Γ is strategy-proof is a consequence of the following arguments:

(i) When Φ is not strategy-proof, there exists $\hat{h} \in H$ such that, for some profiles $\sigma = (\sigma_h)_{h\in H}$ and $\sigma' = (\sigma'_h)_{h\in H}$ in $\mathcal{R}^{\mathrm{std}}$, we have that $\Phi[\sigma_{-\hat{h}}, \sigma'_{\hat{h}}](\hat{h})\, \sigma_{\hat{h}}\, \Phi[\sigma](\hat{h})$. Let $R = (R_h)_{h\in H}$ and $R' = (R'_h)_{h\in H}$ be profiles in $\mathcal{R}^{\mathrm{ego}}$ such that $\sigma(R) = \sigma$ and $\sigma(R') = \sigma'$. Since R and R' are egocentric preference profiles, it follows that $\Gamma[R_{-\hat{h}}, R'_{\hat{h}}]\, P_{\hat{h}}\, \Gamma[R]$. Therefore, the mechanism Γ is not strategy-proof.

(ii) When Φ is bossy, there exists a student $\hat{h} \in H$ such that, for some profiles $\sigma = (\sigma_h)_{h\in H}$ and $\sigma' = (\sigma'_h)_{h\in H}$ in $\mathcal{R}^{\mathrm{std}}$, the following conditions hold $\Phi[\sigma_{-\hat{h}}, \sigma'_{\hat{h}}](\hat{h}) = \Phi[\sigma](\hat{h})$ and $\Phi[\sigma_{-\hat{h}}, \sigma'_{\hat{h}}] \neq \Phi[\sigma]$. Let $R = (R_h)_{h\in H}$ and $R' = (R'_h)_{h\in H}$ be profiles in $\mathcal{R}^{\mathrm{ego}}$ such that $\sigma(R) = \sigma$ and $\sigma(R') = \sigma'$. Moreover, assume that the egocentric preferences $R_{\hat{h}}$ are such that $\Phi[\sigma_{-\hat{h}}, \sigma'_{\hat{h}}]\, P_{\hat{h}}\, \Phi[\sigma]$. Then, it follows form the definition of Γ that that $\Gamma[R_{-\hat{h}}, R'_{\hat{h}}]\, P_{\hat{h}}\, \Gamma[R]$. Hence, the mechanism Γ is not strategy-proof.

On the other hand, assume that Γ is not strategy-proof. Then, there exists $\hat{h} \in H$ such that, $\Gamma[R_{-\hat{h}}, R'_{\hat{h}}]\, P_{\hat{h}}\, \Gamma[R]$ for some profiles of preferences $R = (R_h)_{h\in H}$ and $R' = (R'_h)_{h\in H}$ in $\mathcal{R}^{\text{ego}}$. Since this implies that $\Gamma[R_{-\hat{h}}, R'_{\hat{h}}] \neq \Gamma[R]$, there are two cases of interest:

(i) When $\Gamma[R_{-\hat{h}}, R'_{\hat{h}}](\hat{h}) \neq \Gamma[R](\hat{h})$, it follows from the definition of Γ and $\sigma(R_{\hat{h}})$ that

$$\Phi[(\sigma(R_h))_{h\neq\hat{h}}, \sigma(R'_{\hat{h}})](\hat{h})\ \sigma(R_{\hat{h}})\ \Phi[\sigma(R)](\hat{h}).$$

This implies that the mechanism Φ is not strategy-proof.

(ii) When $\Gamma[R_{-\hat{h}}, R'_{\hat{h}}](\hat{h}) = \Gamma[R](\hat{h})$, it follows from the definition of Γ that

$$\begin{aligned} \Phi[(\sigma(R_h))_{h\neq\hat{h}}, \sigma(R'_{\hat{h}})](\hat{h}) &= \Phi[\sigma(R)](\hat{h}), \\ \Phi[(\sigma(R_h))_{h\neq\hat{h}}, \sigma(R'_{\hat{h}})] &\neq \Phi[\sigma(R)]. \end{aligned}$$

This implies that the mechanism Φ is bossy.

Therefore, Γ is strategy-proof as long as Φ is strategy-proof and non-bossy. □

In the absence of externalities, if we consider mechanisms that are individually rational and strategy-proof for every school choice context, there is a tension between stability and efficiency. On the one hand, $\text{DA}_{(\succ,q)}$ is the only mechanism that is stable and strategy-proof in $\mathcal{R}^{\text{std}}$, but it is not Pareto efficient (see Alcalde and Barberá (1994, Theorem 3) and Ergin (2002)). On the other hand, the mechanism $\text{TTC}_{(\succ,q)} : \mathcal{R}^{\text{std}} \to \mathcal{M}$ which associates with each $\sigma \in \mathcal{R}^{\text{std}}$ the result of the *top trading cycles algorithm* applied to $(S, H, \succ, q, \sigma)$ is Pareto efficient, individually rational, and strategy-proof, but not stable (see Pápai (2000) and Abdulkadiroğlu and Sönmez (2003, Propositions 3 and 4)).

Under weak externalities, Pareto efficiency dominates stability in the search for a mechanism that is strategy-proof and individually rational for all school choice contexts. Indeed, Theorem 1 guarantees that there are school choice contexts in which no stable mechanism defined in $\mathcal{R}^{\text{ego}}$ is strategy-proof, while the following remark shows that for every school choice context there is a mechanism defined in $\mathcal{R}^{\text{ego}}$ that is Pareto efficient, individually rational, and strategy-proof.

Remark 3 (*Efficient, individually rational, and strategy-proof mechanisms)*

The definition of weak externalities guarantees that any Pareto efficient matching of the problem $(S, H, \succ, q, \sigma(R))$ is also Pareto efficient in $(S, H, \succ, R)$. Therefore, as $\text{TTC}_{(\succ,q)}$ is Pareto efficient, individually rational, strategy-proof, and non-bossy in $\mathcal{R}^{\text{std}}$ (see Pápai (2000) and Abdulkadiroğlu and Sönmez (2003)), our Theorem 2 ensures that the mechanism $\text{TTC}^{\text{ego}}_{(\succ,q)} : \mathcal{R}^{\text{ego}} \to \mathcal{M}$ defined by

$$\text{TTC}^{\text{ego}}_{(\succ,q)}[R] = \text{TTC}_{(\succ,q)}[\sigma(R)], \qquad \forall\, R \in \mathcal{R}^{\text{ego}}$$

is Pareto efficient, individually rational, and strategy-proof. □

In school choice problems without externalities, there are mechanisms defined on $\mathcal{R}^{\text{std}}$ satisfying any combination of two properties between stability, strategy-proofness, and non-bossiness. Indeed, independently of the school choice context $(S, H, \succ, q)$, the *school-optimal stable mechanism* is stable and non-bossy (see Afacan and Dur (2017, Proposition 1 and Theorem 1)), the mechanism $\text{DA}_{(\succ,q)}$ is stable and strategy-proof, and the mechanism $\text{TTC}_{(\succ,q)}$ is non-bossy and strategy-proof.[17]

[17]In *marriage markets* without externalities, Kojima (2010, Theorem 1) shows that stability and non-bossiness are incompatible. That is, there are preference profiles such that, regardless of the stable outcome implemented, an agent can misrepresent her preferences to change the situation of another person without being affected (in some scenarios this agent is a woman and in others it is a man). The incompatibility between stability and non-bossiness does not longer hold in classical school choice problems, where only students can misreport preferences.

However, it follows from the existing literature that there are school choice contexts in which no mechanism defined on $\mathcal{R}^{\text{std}}$ is stable, strategy-proof, and non-bossy.[18] Indeed, Roth (1982, Section 6) shows there are specifications of priorities and quotas $(\succ, q)$ such that the mechanism $\text{DA}_{(\succ,q)}$ is bossy, while Alcalde and Barberà (1994, Theorem 3) show that $\text{DA}_{(\succ,q)}$ is the *only* stable and strategy-proof mechanism with domain $\mathcal{R}^{\text{std}}$.

This impossibility result and Theorem 2 guarantee that within the class of myopic mechanisms no one is stable and strategy-proof. Evidently, this property is weaker than the one obtained in Theorem 1.

5. Ergin-acyclicity reconciles stability and strategy-proofness

In the context of classical school choice problems, Ergin (2002) restricts priority structures and quotas to ensure the existence of a stable, strategy-proof, and non-bossy mechanism. We will adapt Ergin's results in order to find necessary and sufficient conditions over $(\succ, q)$ that guarantee the existence of a stable and strategy-proof mechanism defined for any preference profile in $\mathcal{R}^{\text{ego}}$.

Given schools' priorities and quotas $(\succ_s, q_s)_{s\in S}$, an *Ergin-cycle* is constituted of distinct schools $s', s'' \in S$ and students $h', h'', h''' \in H$ such that the following conditions are satisfied:

- *Cycle condition*: $h' \succ_{s'} h'' \succ_{s'} h'''$ and $h''' \succ_{s''} h'$.
- *Scarcity condition*: There are disjoint sets $H_{s'}, H_{s''} \subseteq H \setminus \{h', h'', h'''\}$, with $|H_{s'}| = q_{s'} - 1$ and $|H_{s''}| = q_{s''} - 1$, such that $H_{s'} \subseteq \{h \in H : h \succ_{s'} h''\}$, and $H_{s''} \subseteq \{h \in H : h \succ_{s''} h'\}$.

A vector of priorities and quotas $(\succ, q)$ is *Ergin-acyclic* when it has no Ergin-cycle.

Notice that, in any of the school choice problems described in the proof of Theorem 1 the *cycle condition* is satisfied by schools $\{s_1, s_2\}$ and students $\{h_2, h_3, h_1\}$, because $h_2 \succ_{s_1} h_3 \succ_{s_1} h_1$ and $h_1 \succ_{s_2} h_2$. Also, since $q_{s_1} = q_{s_2} = 1$, the *scarcity condition* is trivially satisfied. On the other hand, in the problem described in Example 1, schools $\{s_1, s_2\}$ and students $\{h_3, h_2, h_1\}$ satisfy the *cycle condition* as $h_3 \succ_{s_1} h_2 \succ_{s_1} h_1$ and $h_1 \succ_{s_2} h_3$, while the *scarcity condition* trivially holds.

Therefore, it is natural to ask whenever the Ergin-acyclicity of $(\succ, q)$ ensures the existence of stable and strategy-proof mechanisms. The following result gives a positive answer to this question.

Proposition 1. *Given $(S, H, \succ, q)$, the following conditions are equivalent:*

(i) *$(\succ, q)$ is Ergin-acyclic.*

(ii) *The stable mechanism $\text{DA}^{\text{ego}}_{(\succ,q)} : \mathcal{R}^{\text{ego}} \to \mathcal{M}$ is strategy-proof.*

Moreover, if $(\succ, q)$ is Ergin-acyclic, then $\text{DA}^{\text{ego}}_{(\succ,q)} : \mathcal{R}^{\text{ego}} \to \mathcal{M}$ is Pareto efficient.

Proof. It follows from Pápai (2000, Lemma 1) that a mechanism $\Phi : \mathcal{R}^{\text{std}} \to \mathcal{M}$ is strategy-proof and non-bossy if and only if it is *group strategy-proof*.[19] Since Ergin (2002, Theorem 1) and Narita (2021) show that $\text{DA}_{(\succ,q)} : \mathcal{R}^{\text{std}} \to \mathcal{M}$ is group strategy-proof if and only if $(\succ, q)$ is Ergin-acyclic, we conclude that $\text{DA}_{(\succ,q)}$ is strategy-proof and non-bossy if and only if $(\succ, q)$ is Ergin-acyclic. Therefore, the equivalence between properties (i) and (ii) follows as a consequence of Theorem 2.

[18] This property can also be obtained as a direct consequence of Theorems 1 and 2.

[19] A mechanism $\Phi : \mathcal{R}^{\text{std}} \to \mathcal{M}$ is *group strategy-proof* when no coalition of students can misrepresent preferences to improve the situation of at least one of its members without hurting others. That is, there is no $H' \subseteq H$ and $(\sigma_h)_{h\in H}, (\sigma'_h)_{h\in H} \in \mathcal{R}^{\text{std}}$ such that: (i) $\Phi[(\sigma_h)_{h\notin H'}, (\sigma'_h)_{h\in H'}] \, \sigma_{h'} \, \Phi[(\sigma_h)_{h\in H}]$ for some $h' \in H'$; and (ii) there is no $h \in H'$ such that $\Phi[(\sigma_h)_{h\in H}] \, \sigma_h \, \Phi[(\sigma_h)_{h\notin H'}, (\sigma'_h)_{h\in H'}]$.

For any profile $R \in \mathcal{R}^{\text{ego}}$, a Pareto efficient matching of $(S, H, \succ, q, \sigma(R))$ is Pareto efficient in $(S, H, \succ, q, R)$. Therefore, as Ergin (2002, Theorem 1) also shows that $\text{DA}_{(\succ,q)} : \mathcal{R}^{\text{std}} \to \mathcal{M}$ is Pareto efficient if and only if $(\succ, q)$ is Ergin-acyclic, it follows that $\text{DA}^{\text{ego}}_{(\succ,q)} : \mathcal{R}^{\text{ego}} \to \mathcal{M}$ is Pareto efficient as long as $(\succ, q)$ is Ergin-acyclic. □

Although $\text{DA}^{\text{ego}}_{(\succ,q)} : \mathcal{R}^{\text{ego}} \to \mathcal{M}$ is stable and strategy-proof only when $(\succ, q)$ is Ergin-acyclic, other stable and strategy-proof mechanisms might exist for some specifications of schools' priorities and quotas compatible with Ergin-cycles. The next result—which adapts Alcalde and Barberà (1994, Theorem 3) to a model with weak externalities—shows that it is impossible.

Proposition 2. *Given $(S, H, \succ, q)$, if there exists a mechanism $\Gamma : \mathcal{R}^{\text{ego}} \to \mathcal{M}$ that is stable and strategy-proof, then it coincides with $\text{DA}^{\text{ego}}_{(\succ,q)}$.*

Proof. Given $R \in \mathcal{R}^{\text{ego}}$, since the problems $(S, H, \succ, q, R)$ and $(S, H, \succ, q, \sigma(R))$ have the same stable outcomes, the following properties hold:

(i) $\text{DA}^{\text{ego}}_{(\succ,q)}$ implements a stable matching of $(S, H, \succ, q, R)$ in which every student gets a seat at her preferred *attainable school* (see remarks at the end of Section 3).

(ii) The students remaining unassigned are the same in all stable matchings of $(S, H, \succ, q, R)$ (see Theorem 2.22 in Roth and Sotomayor (1990)).

Thus, the result follows from analogous arguments to those made by Alcalde and Barberà (1994) to show that $\text{DA}_{(\succ,q)}$ is the only stable and strategy-proof mechanism defined on $\mathcal{R}^{\text{std}}$. □

The following result is a direct consequence of Propositions 1 and 2.

Theorem 3. *Given a school choice context $(S, H, \succ, q)$, there exists a stable and strategy-proof mechanism $\Gamma : \mathcal{R}^{\text{ego}} \to \mathcal{M}$ if and only if $(\succ, q)$ is Ergin-acyclic.*

When all schools have the same priorities, $(\succ, q)$ is trivially Ergin-acyclic. However, in many admission systems the priority structure is determined endogenously by each school. Hence, the absence of Ergin-cycles is unlikely to hold in a wide variety of interesting settings. From this perspective, Theorem 3 reinforces the idea that weak externalities have a profound effect on incentives.

6. Concluding remarks

We analyzed school choice problems with externalities in which students' preferences are egocentric. In this environment, externalities have no effect on the existence of stable matchings. However, from a mechanism design point of view, the situation is different. Even when externalities play a secondary role in students' preferences, they have deep effects on the incentives to reveal information.

Our main results are summarized in Table 1, which details the properties that were lost by the inclusion of weak externalities. In particular, as the conditions that guarantee the compatibility between stability and strategy-proofness seem very restrictive in scenarios in which priorities are school-specific, the use of a Pareto efficient mechanism dominates alternatives based on stability when the focus is on strategy-proofness.

TABLE 1. THE EFFECTS OF WEAK EXTERNALITIES ON INCENTIVES[20]

Mechanism Design in School Choice Problems	Classical	Weak Externalities
$\text{DA}_{(\succ,q)}$ is strategy-proof	✓	×
$\text{DA}_{(\succ,q)}$ is weakly Pareto efficient	✓	×
There always exists a student-optimal stable matching	✓	×
There is a stable and strategy-proof mechanism	✓	×
There is a stable and weakly Pareto efficient mechanism	✓	×
There is a stable, strategy-proof, and weakly Pareto efficient mechanism	✓	×
$\text{DA}_{(\succ,q)}$ is strategy-proof if and only if $(\succ, q)$ is Ergin-acyclic	×	✓
There is a stable and strategy-proof mechanism if and only if $(\succ, q)$ is Ergin-acyclic	×	✓
$\text{TTC}_{(\succ,q)}$ is Pareto efficient, individually rational, and strategy-proof	✓	✓

Since stable and strategy-proof mechanisms may not exist under weak externalities, there are some natural questions about the vulnerability to manipulation of school admission systems that may be of interest for future research:

- How is the manipulability of $\text{DA}^{\text{ego}}_{(\succ,q)} : \mathcal{R}^{\text{ego}} \to \mathcal{M}$ affected by an increase of Ergin-cycles?

  More formally, given priorities and quotas $(\succ, q)$, let $\mathcal{D}_{(\succ,q)} \subseteq \mathcal{R}^{\text{ego}}$ be the collection of preference profiles for which truth-telling is not a weakly dominant strategy when $\text{DA}^{\text{ego}}_{(\succ,q)}$ is implemented. Since $\mathcal{D}_{(\succ,q)} = \emptyset$ when $(\succ, q)$ is Ergin-acyclic (see Proposition 1), it can be interesting to analyze how $\mathcal{D}_{(\succ,q)}$ evolves as the number of Ergin-cycles increases.
- How does the vulnerability to manipulation of a mechanism change when affirmative action policies are considered? (cf., Abdulkadiroğlu and Sönmez (2003), Kojima (2012), Hafalir, Yenmez, and Yildirim (2013), Ehlers et al. (2014), Echenique and Yenmez (2015)).

To compare mechanisms by their vulnerability to manipulation, the techniques developed by Pathak and Sönmez (2013), Chen et al. (2016), and Bonkoungou and Nesterov (2021, 2022) may be useful (cf., Chen and Kesten (2017), Dur et al. (2022)).

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[20]With a slight abuse of notation, the mechanisms $\text{DA}^{\text{ego}}_{(\succ,q)}$ and $\text{TTC}^{\text{ego}}_{(\succ,q)}$ are labeled as their counterparts for classical school choice problems ($\text{DA}_{(\succ,q)}$ and $\text{TTC}_{(\succ,q)}$, respectively).

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